

Returns to Scale, Productivity, and the Role of Computer Software

Evidence from the UK

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Roadmap

Motivation

Theory

Empirics

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Motivation

What are Returns to Scale?

$$= \underbrace{\frac{\partial y}{\partial C} \frac{C}{y}}_{\text{Elasticity of output to costs.}} = \frac{AC}{MC}.$$

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> 1 reflect faster growth in outputs than inputs (Basu 2008).

Why do Returns to Scale matter?

1. Tightly linked to productivity and firm survival (Gao and Kehrig 2020).
2. It describes the long-run productivity characteristics of an industry.
3. Tells us about production function & extent of imperfect competition.
 - What happens when a market expands? (Baqae and Farhi 2020)
 - Changes the response of firms to policy shocks (Basu and Fernald 1996).
 - Important for antitrust regulation.

Literature

- RTS theory: (Feenstra 2003; Hall 1988; Kee 2002; Ruzic and Ho 2019).
- RTS estimation: (Basu and Fernald 1996; Harris and Lau 1998; Oulton 1996).
 - UK: ≤ 1 in manufacturing up to 1990.
- Impact of software: (De Ridder 2019; Lashkari et al. 2019).

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$$\pi(y) = p(y)y - c(y) \implies \quad = \mu(1 - s_\pi)$$

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Alternatively, from cost-minimisation: [Derivation](#)

$$= \nu(1 + s_\phi)$$

This highlights a productivity puzzle:

- $\uparrow \rightarrow \uparrow$ productivity required for firm to survive (Gao and Kehrig 2020).
- However, more productive firms are larger $\rightarrow \downarrow s_\phi \rightarrow \downarrow$.
 - Intuition: \uparrow productivity $\rightarrow \uparrow y$ and shifts costs curves. Firm moves along new cost curve to point where \mathcal{AC} and \mathcal{MC} are closer.

Elasticities & Returns to Scale

It is straightforward to show that:

$$\text{Returns to Scale} = \text{Sum of output elasticities}$$

These elasticities are what we want to estimate.

Software

Software scales down costs, by making it cheaper to replicate tasks. However, it is associated with a fixed cost to adopt (De Ridder 2019; Kariel 2021).

Hypothesis: adoption of computer software should *raise* returns to scale, by allowing firm output to grow faster than inputs.

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Data

- ARDx data from ONS.
- Approx. 50,000 firms per year, 1998 - 2014.
- Essentially a census for large firms, survey for small firms.
- Covers around 11 million workers.
- Capital stock: PIM on investment data; allocate national capital stock.

Estimation

$$y_{it} = z_{it} + \alpha k_{it} + \beta l_{it} + \gamma m_{it} + \epsilon_{it}$$

where α is the output elasticity we require to obtain returns to scale.

Classic **endogeneity problem**: cannot observe productivity z_{it} , which affects optimal input factor choices.

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Control function approach helps alleviate this problem (Akerberg et al. 2015; Levinsohn and Petrin 2003; Olley and Pakes 1996). [More detail](#)

Mapping theory to data

Technically, we observe **revenue** $P_{it}Y_{it}$, not **output** Y_{it} . Estimated coefficients are **revenue elasticities**, not **output elasticities**.

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we can multiply the **markup** by **revenue elasticities** to obtain **output elasticities**. The **markup** is estimated from:

$$\mu = \frac{m}{m}$$

is the ratio of the elasticity of output to materials inputs, divided by the materials share in revenue.

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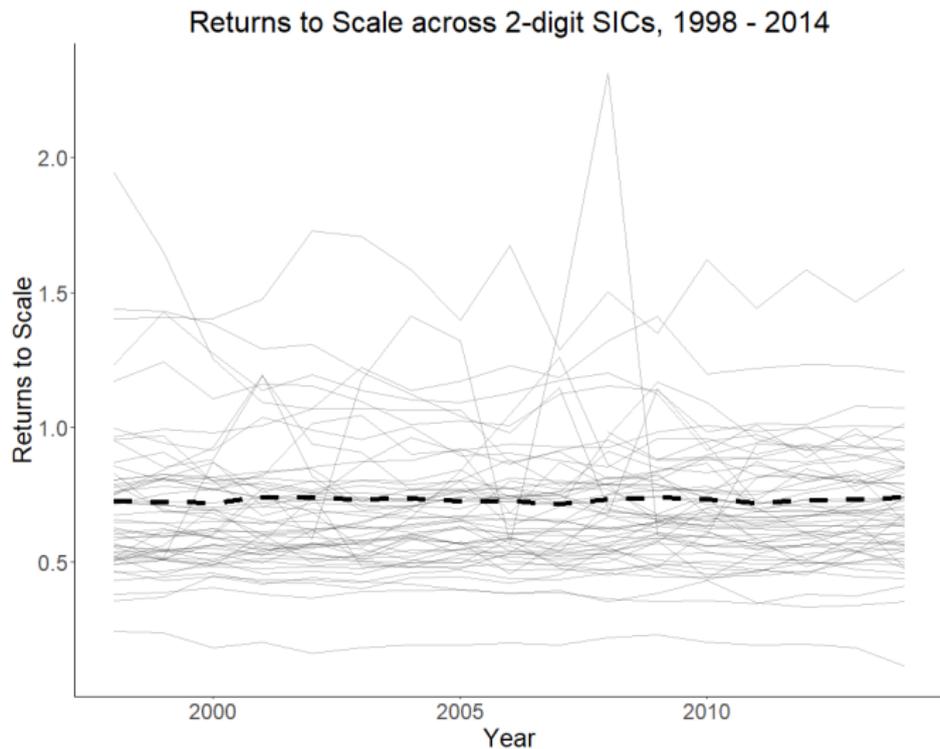
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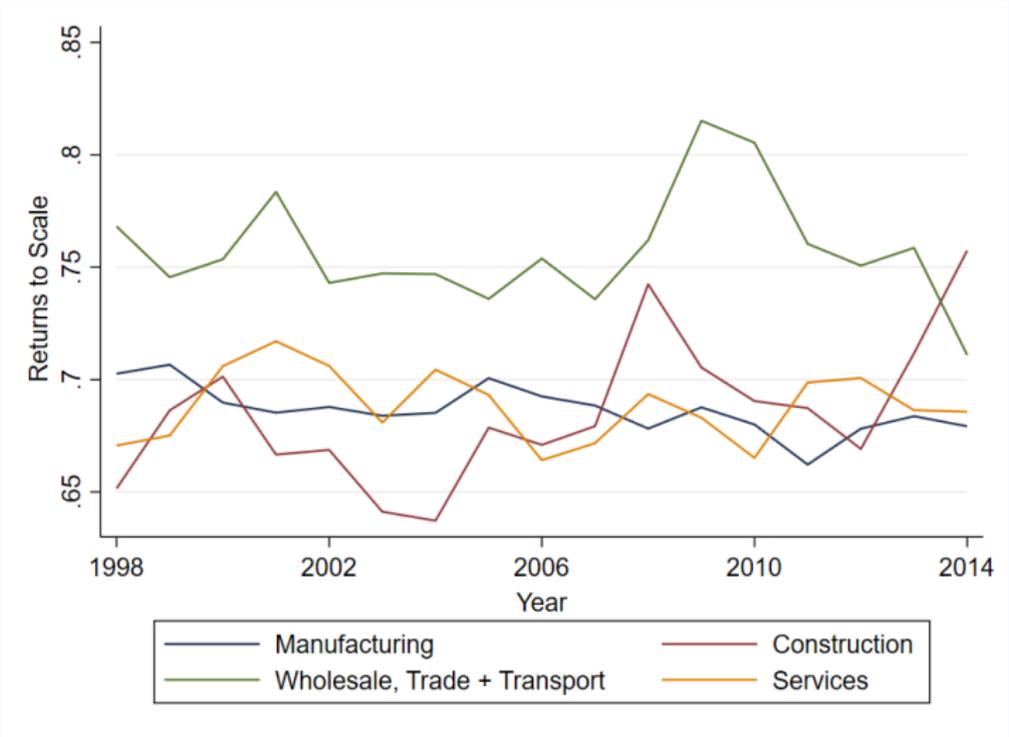
Results

Returns to Scale in the UK



Source: ARDx from ONS

Returns to Scale in the UK



Returns to Scale and Productivity

Table: Regression: Returns to Scale and Log Productivity

<i>Dependent variable: Returns to Scale</i>				
Log TFP	-0.025*** (0.005)	-0.026** (0.005)	0.031 (0.019)	0.093** (0.033)
<i>N</i>	901	901	901	901
2-digit SIC FE:			✓	✓
Year FE:		✓		✓

Estimates statistically significant at levels of 1%: ***, 5%: **, 10%: *. Robust standard errors clustered at the level of the 2-digit SIC.

The Role of Software

Table: Regression: Returns to Scale and Computer Software

	<i>Dependent variable: Returns to Scale</i>			
Software Intensity	-3.367 (6.513)	-3.270 (6.816)	2.403*** (0.514)	2.719** (0.790)
<i>N</i>	820	820	820	820
2-digit SIC FE:			✓	✓
Year FE:		✓		✓

'Software Intensity' is share of computer software in revenue. Estimates statistically significant at levels of 1%: ***, 5%: **, 10%: *. Robust standard errors clustered at the level of the 2-digit SIC.

Conclusions

1. Estimate RTS across UK economy:
 - Decreasing RTS.
 - Significant heterogeneity.
 - Slight rise over time.
2. Estimate RTS with non-constant markups.
3. Relationship between RTS & productivity is nontrivial: negative *between* industries; positive *within* industries.
4. Software is associated with higher RTS.

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Returns to Scale derivation

Cost minimising firms solve:

$$C := \min_{K,L} wL + rK \quad \text{s.t.} \quad y \geq zF(K, L) - \phi.$$

The solution yields:

$$C = \lambda y \left(\varepsilon_{yL} + \varepsilon_{yK} \right)$$

Applying Euler's homogeneous function theorem, we get:

$$\begin{aligned} C &= z\lambda y \left(\frac{\partial y}{\partial L} \frac{L}{y} + \frac{\partial y}{\partial K} \frac{K}{y} \right) = z\lambda \left(\frac{\partial y}{\partial L} L + \frac{\partial y}{\partial K} K \right) \\ &= \lambda \nu (y + \phi) \end{aligned}$$

It follows that the ratio of average to marginal costs is:

$$\frac{\mathcal{AC}}{\mathcal{MC}} = \frac{\lambda \nu (1 + s_\phi)}{\lambda} = \nu (1 + s_\phi)$$

Control Function Approach I

Taking logarithms, we get:

$$y_{it} = \alpha_0 + \alpha_K k_{it} + \alpha_L l_{it} + \alpha_M m_{it} + \epsilon_{it}.$$

where $\ln z_{it} = \alpha_0 + \epsilon_{it}$.

Olley and Pakes (1996): timing of input choices; investment is a proxy for unobserved productivity shocks. Split up unobserved residual

$\epsilon_{it} = \omega_{it} + \eta_{it}$, where ω_{it} is anticipated and η_{it} is an ex-post shock.

Control Function Approach II

Assumptions:

1. **Information Sets:** include current and past productivity shocks $\{\omega_{i\tau}\}_{\tau=0}^t$, but firms know nothing about future shocks.
2. **First-Order Markov Shocks:** productivity shocks follow a First-Order Markov Process, so $\omega_{it} = \mathbb{E}(\omega_{it}|\omega_{i,t-1}) + \nu_{it}$.
3. **Timing of Input Choices:** previous period $i_{i,t-1}$ determines future capital k_{it} , whereas labour is chosen contemporaneously.
4. **Scalar Unobservable:** investment decisions $i_{it} = f_t(k_{it}, \omega_{it})$ have just one scalar unobservable ω_{it} .
5. **Strict Monotonicity:** investment decisions are strictly monotonic in the scalar unobservable ω_{it} , so $i_{it} = f_t(k_{it}, \omega_{it})$.

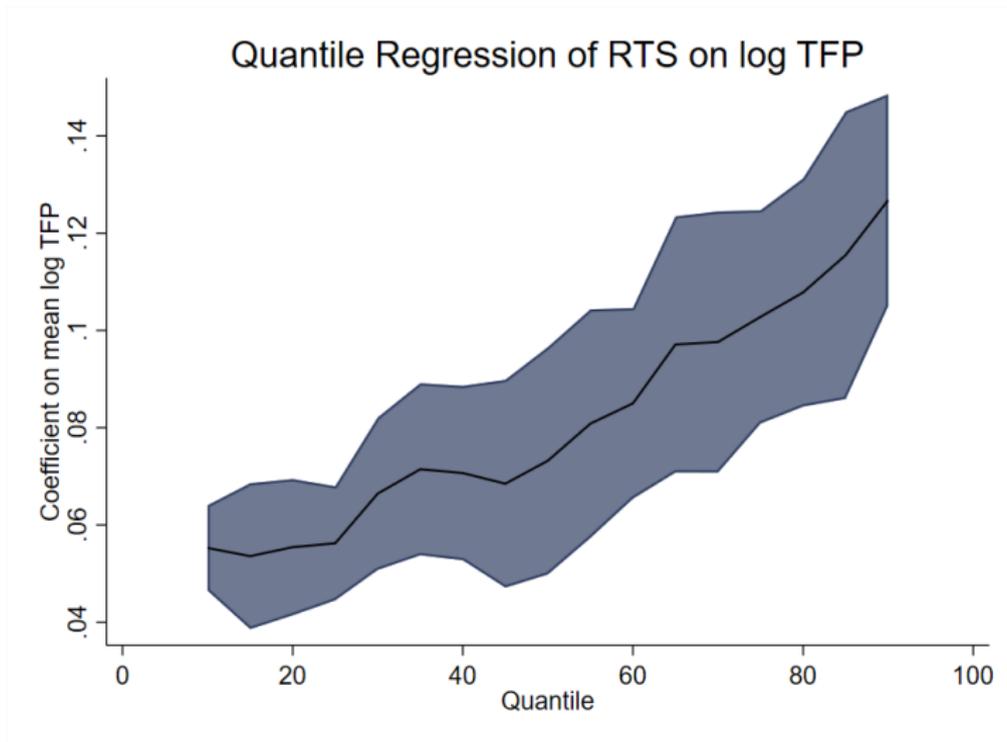
As i_{it} is strictly monotonic in unobserved anticipated shock, this function is inverted:

$$y_{it} = \alpha_0 + \alpha_k k_{it} + \alpha_l l_{it} + \alpha_m m_{it} + f_t^{-1}(k_{it}, i_{it}) + \eta_{it},$$

and the inverted function is approximated by a polynomial in k_{it}, i_{it} .

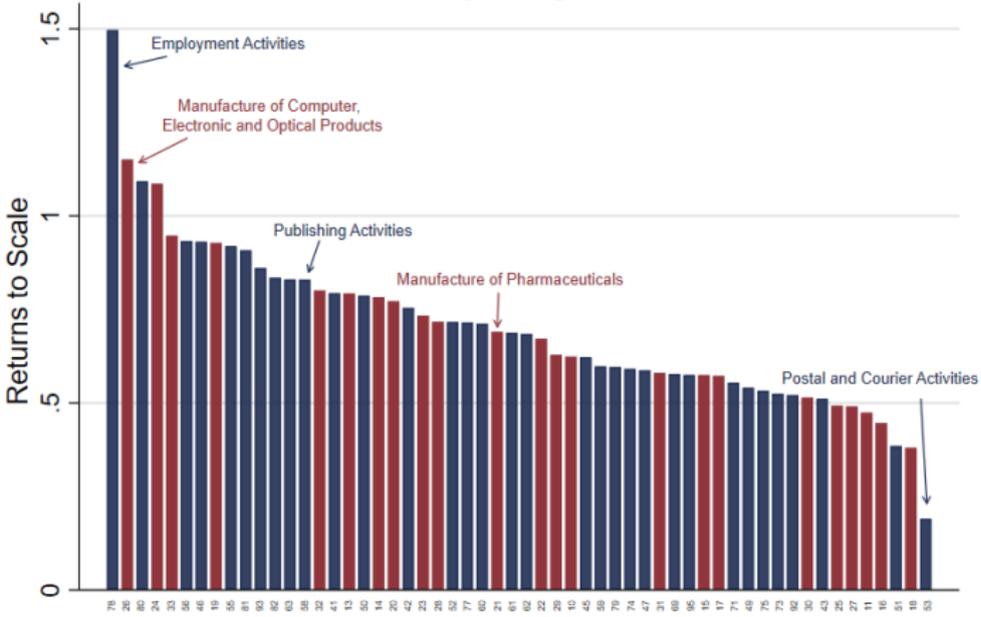
[Return](#)

Quantile Regression RTS on log TFP



Returns to Scale Heterogeneity

Returns to Scale by 2-digit SIC, 1998 - 2014



Source: ARDx from ONS

Returns to Scale by Macro Sector

Table: Returns to Scale Estimates using Levinsohn and Petrin (2003)

	<i>Manufacturing</i>	<i>Construction</i>	<i>Wholesale, Trade + Transport</i>	<i>Services</i>
μ	0.740	0.795	0.981	0.873
ζ	0.928	0.860	0.771	0.788
	0.686	0.684	0.757	0.688

Estimated RTS using Cobb-Douglas production function, Levinsohn and Petrin (2003) control function method, with gross output for revenue elasticities and markup estimation.